

# How WBGF World Ranking is Calculated

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The WBGF Backgammon World Ranking relies on the computation of “Performance Points” (PP) for players who have competed in eligible tournaments. Tournaments always have a “primary”, or Championship Main event, which may have Consolation and Last Chance rounds. Intermediate Main and Super Jackpot events, subject to certain criteria, also qualify for Performance Points.

## Performance Points for Main, Main Consolation, and Main Last Chance

In all cases, the result for a player in a tournament event is simply the rank they attain, which may possibly be shared with other players. In this section, we consider just the Main events. Calculations for Super Jackpot and Intermediate events are similar, and are discussed later.

Each event that they rank in yields a PP score calculated as follows.

Let  $G$  be the tournament grade, calculated as

$$G = \frac{\text{Main entry fee (in €)} + \left( \frac{\text{Main added money (€)}}{\text{number of Main entrants}} \right)}{100}$$

Note that only added money for the Main event counts – added money for intermediates, doubles, etc. is ignored. The grade  $G$  is capped at 5 (achieved when the Main entry fee is €500, or less if there is added money). Bonus grades are no longer awarded, but when they were, they were added to  $G$  after capping. Previously, the Euro exchange rates for historical results were taken from <https://www.lollandsbank.dk/valutakurser/> but it is unclear how frequently this data was refreshed. In the current system, the mid-market rates on the last day of the tournament, courtesy of [fixer.io](https://fixer.io), are used.

Let  $F$  be the Format Factor of the event, assigned as follows:

Event	If the primary tournament only has a Main event	If the primary tournament has Main and Consolation only	If the primary tournament has Main and Last Chance only	If the primary tournament has all of Main, Consolation, and Last Chance
Main	1	0.75	0.9	0.7
Consolation		0.25		0.2
Last Chance			0.1	0.1
Intermediate Main	0.3	0.3	0.3	0.3
Super Jackpots	1/3	1/3	1/3	1/3

Let  $k = 0.75$ . This is a constant value in the spreadsheet used originally to compute the WR formula. The cell reference is 'maskinrum NY'!B22, and is labelled “Stigning i %” (% increase).

Let  $t = 10$ . This is a constant value in the spreadsheet used originally to compute the WR formula. The cell reference is 'maskinrum NY'!C12, labelled “gang points med factor” (time points factor?).

Define the tournament size scaling function  $s(n)$  by

$$s(n) = \begin{cases} n, & n \leq 128 \\ 128(H_n - H_{128} + 1), & n > 128 \end{cases}$$

where  $H_n$  is the  $n^{\text{th}}$  harmonic number  $H_n = \sum_{t=1}^n \frac{1}{t}$ .

Let  $N$  be the number of entrants in the event under consideration (Main, Consolation, or Last Chance), and  $N_{\text{main}}$  the number of entrants in the Main event specifically.

Next, define the rank reward helper function  $R(r)$  for an event with  $N$  entrants recursively by

$$R(N) = 1,$$

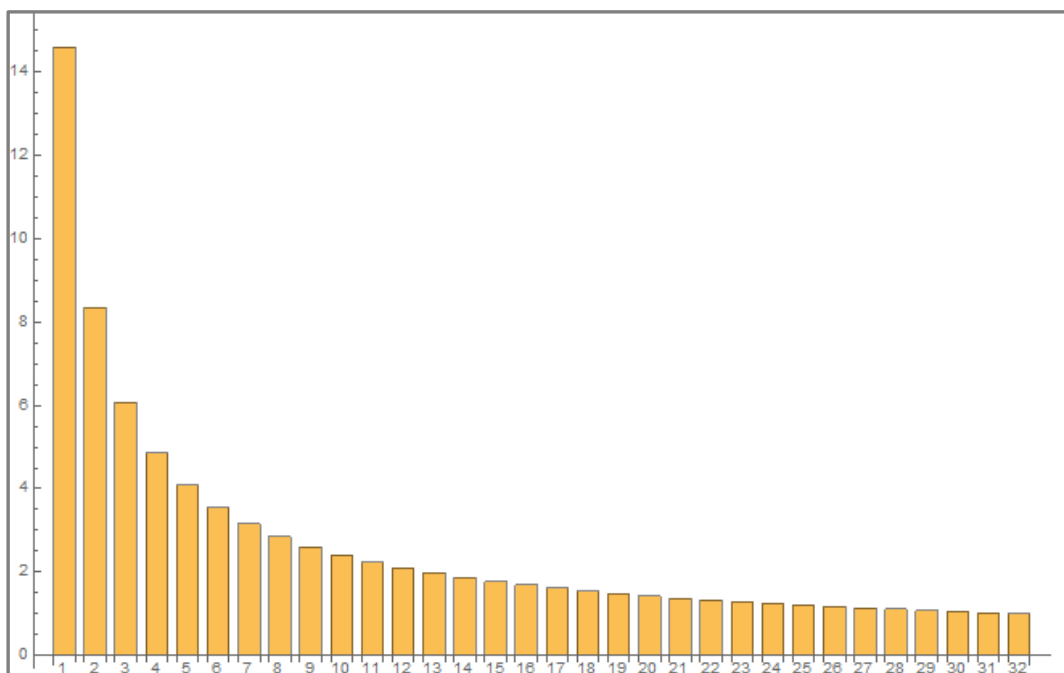
$$R(r) = \left(1 + \frac{k}{r}\right) R(r + 1)$$

The solution of this recurrence relation is

$$R(r) = \frac{\Gamma(N + k) \Gamma(r)}{\Gamma(N) \Gamma(k + r)}$$

where  $\Gamma(x)$  is the gamma function  $\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$ , which obeys  $\Gamma(x + 1) = x\Gamma(x)$  and coincides with  $(x - 1)!$  when  $x$  is a positive integer.

The rank reward function strongly favours those achieving the highest ranks. For example, in a 32-player tournament with no ties, ranks achieved are rewarded as shown in the chart below:



The winner of a 32-player tournament gains 14.6 times as many WR points as the person who came last. For a 64-player event, the ratio is 24.6; for a 128-player event, it is 41.3.

The *provisional* performance points value for a player at rank  $r$  is now defined as the product of  $G$ ,  $F$ ,  $t$ ,  $s(N_{main})$ , and  $R(r)$ . This is normalised by dividing through by  $\sum_{r=1}^N R(r)$ , a sum that can be shown to be equal to

$$\frac{1}{k-1} \left( \frac{\Gamma(N+k)}{\Gamma(N)\Gamma(k)} - N \right)$$

Finally, if  $m$  players are tied at rank  $r$ , the performance points awarded to each of those players is the arithmetic mean of the provisional performance points for ranks  $r$  through to  $r+m-1$ .

Putting this all together, the full formula for the Performance Points  $PP_r$  awarded to the player who ranked in  $r^{\text{th}}$  place in an event, tied with  $m-1$  other players, is equal to

$$\underbrace{PP_r}_{m \text{ tied}} = G \cdot F \cdot t \cdot s(N_{main}) \cdot \frac{\Gamma(k)\Gamma(N+k)}{m(\Gamma(N+k) - \Gamma(k)\Gamma(N+1))} \left( \frac{\Gamma(r)}{\Gamma(r+k-1)} - \frac{\Gamma(r+m)}{\Gamma(r+m+k-1)} \right)$$

This simplifies in the case  $m=1$  to

$$\underbrace{PP_r}_{no \text{ ties}} = G \cdot F \cdot t \cdot s(N_{main}) \cdot \frac{(k-1)\Gamma(r)\Gamma(k)\Gamma(N+k)}{\Gamma(r+k)(\Gamma(N+k) - \Gamma(k)\Gamma(N+1))}$$

## Performance Points for Super Jackpot Events

The grade,  $G$ , is calculated using the same formula but with reference to the entry fee, number of players, and added money for the Super Jackpot event, rather than the Main.

Note that  $F = 1/3$  always.  $t$  and  $k$  are unchanged.

The PP for Super Jackpot events is calculated using the same formula as for Main events, replacing both  $N$  and  $N_{main}$  with the number of entrants in the Super Jackpot.

## Performance Points for Intermediate Events

Only an Intermediate event's Main event (not consolation, etc.) qualifies for WR points. The grade,  $G$ , is calculated using the same formula with reference to the entry fee of the Main, and with any added money disregarded. That is,

$$G = \frac{\text{Main entry fee (in €)}}{100}$$

Note that  $F = 0.3$  always.  $t$  and  $k$  are unchanged.

The PP for Intermediate events is calculated using the same formula as for Main events, replacing both  $N$  and  $N_{main}$  with the number of entrants in the Intermediate event.

## WR Points

World Ranking points for a player in an event starts equal to the PP and declines linearly over the course of 3 years, starting from the final date of the event, halting at zero. Leap days are ignored.

$$WR = PP \cdot \max(0, 1 - \textit{age}/1095)$$

where *age* is the number of days from the last day of the tournament to today's date.

## World Ranking

For every player, WR is summed across all events in all tournaments attended. In practice, only the last three years' of tournaments need be considered.

The World Ranking is the ranking of all players' WR sums.

## National Ranking

Each player's nationality is held within the system. The national ranking of a country is the sum of WR for all players from that country.